GUIDED PROBLEM SOLVING FOR FIRST YEAR TERTIARY STUDENTS

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Although over the last few years there has been much effort devoted to introducing problem solving and modelling into our teaching of mathematics, up to now there has been little progress on this at the tertiary level. At this level there appears to be just too many problems to overcome and little research has been undertaken. In this paper reasonably efficient problem solving guided approach that addresses some of the issues is proposed and some accompanying resource material presented.

INTRODUCTION

Teaching mathematics using a problem solving approach has been with us for a long time and much has been written on the subject but at least at the tertiary level not much has been achieved in the way of implementing a problem-based program. A serious obstacle may be that it is difficult to integrate problem solving in subjects (such as the typical first year tertiary mathematics) that are necessarily content driven. In this paper a solution is presented - a problem guided approach where problems are used to motivate ideas and theory.

THE PROBLEMS

Here are some of the well-known symptoms associated with the teaching of mathematics at the first-year tertiary level:

1. High attrition rates in many mathematics units.

2. Decay in numbers of students actually attending lectures and tutorials over the semester or term.

3. Many students not doing enough work in their own time (homework or assignments).

4. Students' lack of motivation.

5. A low pass rate or "lower standard" in many mathematics units.

6. Mathematics courses typically having small enrolments.

Although there are notable exceptions, these symptoms appear to be somewhat independent of the style of the lecturer (within reason) and even of the country (see, for example, MS2000 Committee, 1991) so it seems that something else is wrong. Often students themselves are blamed and while it cannot be denied that there has been an overall drop in the quality of students attending tertiary institutions - they are coming in record numbers and the overall quality must therefore drop - as their teachers we must work with what we have, not with what we wish we did have. We must make some changes now and there are many possibilities. Here are some problem areas that have been and continue to be addressed by many teachers at many institutions.

1. The lecture/tutorial system. This doesn't seem to work for many (most?) students studying mathematics. Mathematics can[not] be learned by watching someone else do it correctly" (MS2000 Committee,1991).

2. The lack of preparation of incoming students. Many teachers at the tertiary level are concerned that they are continually seeing a drop in the level of preparation of students.

3. The subject matter of mathematics subjects often does not relate to students' other subjects.

These are obvious problems and there are some obvious "cures"; for example, having students in smaller groups and/or forming special learning centres, implementing bridging mathematics subjects, splitting groups of students up into discipline-based groups. But there are other, more serious and fundamental, problems, that are not so often tackled though they are just as well-known.

4. There has been and continues to be an undue emphasis on purely manipulative techniques and associated "standard" problems that can be done by having the students follow a "recipe". And while it is definitely important that students become proficient at manipulative skills it is obvious that simply teaching them in association with the "standard" problems doesn't work. Many (most?) students will be unable to cope if they are given a slightly novel problem that involves just relatively simple techniques.

5. Mathematics is traditionally taught as a series of seemingly unrelated topics, usually in different subjects (Precalculus, Calculus, Vectors and Matrices, Probability and Statistics) yet we wish the students to ultimately be able to draw on all tools as needed.

6. The problem mentioned as problem 2 actually goes much deeper than is often recognized. Many students lack skills as "elementary" as the ability to work with fractions and square root and .... This deficit becomes especially clear when students are required to use elementary techniques in a novel problem (as mentioned as problem 4) but even a purely mechanical question dissociated from any practical problem can cause trouble for many students.

7. The problem mentioned as problem 3 is actually a little too specific - the real problem is that there is often an almost complete lack of any motivational material, course specific or otherwise.

8. Textbooks are mostly all the same (and take an approach that only makes the above problems worse).

PROBLEM SOLVING AS A CURE

At the tertiary level, a problem solving approach to teaching mathematics seems to offer cures for each of the problems listed above. The "cures" are numbered to indicate the respective problems that they address.

1. Problem solving is best done in smaller groups (workshops) and thus "forces" (to some extent) teachers to abandon the lecture/tutorial system.

2. Problem solving forces students to develop and use all their mathematical skills together. Thus it provides a mechanism for students to work on their basic or elementary skills (problems 2 and 6) as well as the new material they are supposed to learn.

3. Problem solving should increase and maintain motivation provided that the students can relate to the problems studied (problems 3 and 7).

4. By using a problem solving approach we obviously help students in learning to use mathematical tools to solve problems (problem 4). And if we are careful we can still cover the range of techniques and mechanical skills that they would be presented with (and fail to learn!) in the more traditional approach.

5. Problem solving forces an integrated approach to the teaching of mathematics (problem 5). It is almost always the case that a problem involves the use of techniques from a range of traditional topics.

6. If problem solving is to be used, new textbooks and other resource material must be developed for students and teachers.

Given the above advantages of the problem solving approach, it may seem somewhat surprising to find that most teachers at the tertiary level are reluctant to try it. Indeed, many teachers are reluctant to try any new ideas at all and instead adopt the traditional approach (lecture/tutorial system with one of the standard textbooks that does not use a problem solving approach). There are many reasons for this:

1. Tertiary mathematics subjects, especially the all-important service subjects in Science, Applied Science and Engineering are necessarily content driven and teachers at this level are under considerable pressure to cover a large amount of required material. Unfortunately, a standard problem solving approach is more demanding in terms of the main resource, time, than is the traditional approach. Students simply do not have the luxury of being able to discover and experiment for themselves. A solution is presented in the following section.

2. There is a serious lack of resource material. Current textbooks nearly all take the traditional approach and are more-or-less all the same. Potter (1990) points out textbook publishers may be partially to blame. A solution is presented in the following section.

3. There is a general lack of knowledge and experience as to what techniques for teaching problem solving actually work at the tertiary level.

4. In spite of much rhetoric, there is still very little incentive for teachers at the tertiary level to experiment and improve their teaching. Promotion is based primarily on research, not teaching. In fact there appears to be so little incentive for experimentation that about the "best" that many teachers at the tertiary level have so far managed to do is introduce some assignments or tasks that involve the use of problem solving techniques to some extent while keeping the rest of the subject more-or-less the same.

THE PROBLEM-GUIDED APPROACH

In this section we present a program and related support material for an approach to teaching first year tertiary mathematics that employs an essentially problem solving approach while taking into account the essentially content driven nature of this subject. The basic idea is to use what may be called a problem guided technique and relies very heavily on a textbook (1992), which was written especially for the subject. The book consists of two parts - a problem solving part and an overlapping theory part.

The problem solving part of the book consists of a set of carefully chosen problems (applications) that are presented one after another in a sequence designed to introduce key ideas in a logical order ("elementary" to "more advanced"). There are many associated problems for the students to do, too. Together these applications are designed to form a more-or-less complete book on their own, but, and this is what is designed to satisfy the content driven nature of the subject, interspersed within them is another set of notes that contains the relevant theory and skills and techniques (and many associated drill exercises) that arise in or are needed for their solution. Ideally this "theory" sub-book is intended to tag along as needed. Sometimes sections appear at the end of an application (if the solution is sufficiently short); other times they may pop-up in the middle of the solution to an application because some new theory or idea needs developing. In reality, however, the sub-books were written together so that ideas are developed in a logical order and, in fact, the "theory" sub-book often was designed before the associated application in the "problem solving" sub-book was found.

So the whole book is therefore formed by interspersing two intimately related sub-books - "problem solving" and "theory". It is sometimes difficult to separate the two and they are necessarily heavily interdependent, but the logical difference is usually clear. They are clearly distinguished from one another in the entire set of notes by having them printed in different colours or on different coloured backgrounds.

An example of the problem - guided approach will help make the idea clear. The topic is L'Hopital's rule and the application used to introduce the idea was actually used earlier in the book to motivate the whole notion of a limit - quite a large section of work followed where some elementary techniques (using a calculator, graphical techniques, use of "algebra tricks", etc) were presented together with the use of a symbolic manipulator and a discussion of the Squeeze Theorem. Now the problem is tackled again. In the book the application and its solution were printed on coloured paper to separate them from the associated theory. Here this material has been placed in a box. Some details have been omitted and additional comments added (in italics).

Example (taken from Carr (1992))

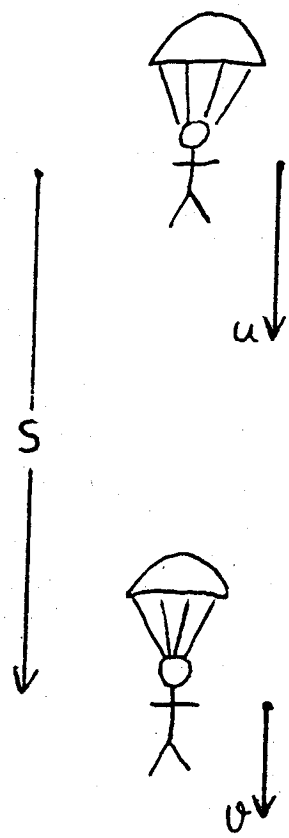
Recall Application 310:

This does not mean that there have been 310 applications - all items (Applications, Definitions, Examples, etc) are numbered sequentially.

APPLICATION 310. After opening a parachute, the distance s travelled by a parachutist of mass m can be shown (see **449** - this refers to a previous problem) to be related to the velocity v by

$$(*) \qquad s = \frac{m}{2k} \ln \left(\frac{mg - ku^2}{mg - kv^2} \right)$$

where k is the drag constant and u the speed of the parachutist when the parachute is opened (v can be greater than or equal to u or less than u). This should be a generalization of the familiar physics formula relating velocity with displacement without resistance:



$$v^2 = u^2 + 2gs$$

$$(**) \qquad = \qquad s = \frac{v^2 - u^2}{2g}$$

Check that (*) simplifies to (**) if k = 0.

SOLUTION 512. Unfortunately, we can't plug k = 0 directly into (*) (why?), so what we really have to show is

$$\lim_{k \to 0^+} \frac{m}{2k} \ln \left(\frac{mg - ku^2}{mg - kv^2} \right) = \frac{v^2 - u^2}{2g}$$

(Continued in Solution 520)

The limit in Solution 512 cannot be done (by hand) using any of the methods covered previously (Sections 12.3.8.1-4), so we need something new -

12.3.8.5. Evaluation of limits using L'Hopital's rule

In earlier parts of Section 12.3.8 we looked at some simple methods for evaluating limits (that involved, for the most part, the use of graphs, etc, on a computer, simple algebra tricks or the squeeze theorem). But there are still many limits (like the one in Solution 512) that we cannot evaluate this way. In this section we will look at a very useful result that will help us further.

THEOREM 513. L'HOPITAL'S RULE for the indeterminate forms 0/0 and ∞/∞.

Let x_0 be a real number, $+\infty$ or $-\infty$, and let f and g be two differentiable functions so that

$$\lim_{x\to x_0} f(x) = \begin{cases} 0 \\ \pm\infty \end{cases} \quad \text{and} \quad \lim_{x\to x_0} g(x) = \begin{cases} 0 \\ \pm\infty \end{cases}$$

(ie, 0/0 or $\pm\infty/\pm\infty$)

If

$$\lim_{x\to x_0} \frac{f'(x)}{g'(x)} = L$$

(where L is a real number, $+\infty$ or $-\infty$) then

$$\lim_{x\to x_0} \frac{f(x)}{g(x)} = L.$$

PROOF

The "fake proff" and the formal proof using the Mean Value Theorem are presented - details omitted.

Some examples showing how to use the theorem and some "drill-type" exercises are given (3-4 pages) - details omitted.

SOLUTION 512 (continued) 520.

To verify this, note that we can write

$$\lim_{k\to 0^+} \frac{m}{2k} \ln\left(\frac{mg - ku^2}{mg - kv^2}\right) = \frac{m}{2} \lim_{k\to 0^+} \frac{\ln\left(\frac{mg - ku^2}{mg - kv^2}\right)}{k}$$

with

$$\lim_{k\to 0^+} \ln\left(\frac{mg - ku^2}{mg - kv^2}\right) = \ln(1) = 0 \quad \text{and} \quad \lim_{k\to 0^+} k = 0.$$

So L'Hopital's rule applies:

$$\lim_{k\to 0^+} \frac{m}{2k} \ln\left(\frac{mg - ku^2}{mg - kv^2}\right)$$

$$= \frac{m}{2} \lim_{k\to 0^+} \frac{\ln\left(\frac{mg - ku^2}{mg - kv^2}\right)}{k} \qquad (0/0)$$

$= \ldots$ (Details omitted)

$$= \frac{v^2 - u^2}{2g}$$

as expected.

NOTE 521. Derive is able to evaluate most limits that use L'Hopital's rule (it may not acually use the rule, but the effect is the same).

For example, the above limit could have been determined using Derive:

SOLUTION 512 (continued) 522.

To verify this use Derive:

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Author lim(m/(2k) ln((mg-ku^2)/(mg-kv^2)),k,0,1) ↵
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$$1: \lim_{k \to 0+} \frac{m}{2\ k} \text{LN} \left[\frac{m\ g - k\ u^2}{m\ g - k\ v^2} \right]$$

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Simplify ↵
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$$2: - \frac{u^2 - v^2}{2\ g}$$

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So $\lim_{k \to 0^+} \frac{m}{2k} \ln\left(\frac{mg - ku^2}{mg - kv^2}\right) = \frac{v^2 - u^2}{2g}$ as expected.

COMMENTS ON THE PROGRAM

There are some interesting points and some difficulties with the program that may be apparent even from the above small example.

The first is that computer programs (including spreadsheets, graphical packages, numerical equation solvers and symbolic manipulators) are used very often as mathematical tools (not just as teaching aids) and students are expected to learn how to use them, too.

Another point is that some topics (the above one is not one) are introduced in a fairly non-traditional manner. Definite integrals, for example, are first evaluated numerically (using a computer, for example) rather than using the Fundamental Theorem of Calculus (and the topic may actually be best done before differentiation, as Courant (1938) does). The exponential function is defined by

$$\exp(x) = \lim_{n \to \infty} (1 + x/n)^{1/n}$$

rather first defining the natural logarithm via a definite integral. Probability is introduced in the context of an hypothesis test (though it's not called that, of course).

One difficulty (and this is a problem with any problem solving approach) is that we must rely on students having a sufficiently good background in other non-mathematical subjects. In the above example the given equation was derived in a separate problem that relied on the use of Newton's $F = ma$ law.

A more serious problem with the approach is that because theory is so dependent on the applications and appears only as it is needed, it is difficult to use the notes as a reference book. It is difficult even to read the notes in small pieces. Often an idea becomes clear only when a whole slab is read. The first impression can be that the work appears disorganized and difficult to follow. A serious flaw for a textbook for first year students ! - and it does require considerable perseverance and careful reading.

The system has been developed and trialled by the author over the past few years and currently the notes contain all the relevant material found in a typical first year "calculus/linear algebra/probability & statistics" sequence. Limited initial trials using the approach seemed to indicate that there was no dramatic, if any, improvement in students' performance over the year. However, there seemed to be no reasons for not pursuing the program further and the current version of the notes is complete enough for a full trial. This is in progress at the moment, but unfortunately any effects will necessarily be difficult to detect because the program is being followed by a whole group of new students with no control group for comparison (the introduction of the Victorian Certificate of Education makes it impossible to sensibly compare this year with previous years). However, it is probably not worthwhile to attempt to quantify the approach yet. There are still some serious design problems in the notes that need to be rectified.

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